

# A Class File for AIP

## The parameter section

Frank Mittelbach

February 9, 2009

### **Contents**

# Chiral Odd Generalized Parton Distributions in Position Space

A. Mukherjee\*, D. Chakrabarti<sup>†</sup> and R. Manohar\*

*\*Department of Physics, Indian Institute of Technology, Powai, Mumbai 400076, India.*

*<sup>†</sup>Department of Physics, Swansea University, Singleton Park, Swansea, SA2 8PP, UK*

**Abstract.** We report on a calculation of the chiral odd generalized parton distributions (GPDs) for non-zero skewness  $\zeta$  in transverse and longitudinal position spaces by taking Fourier transform with respect to the transverse and longitudinal momentum transfer respectively using overlaps of light-front wave functions (LFWFs).

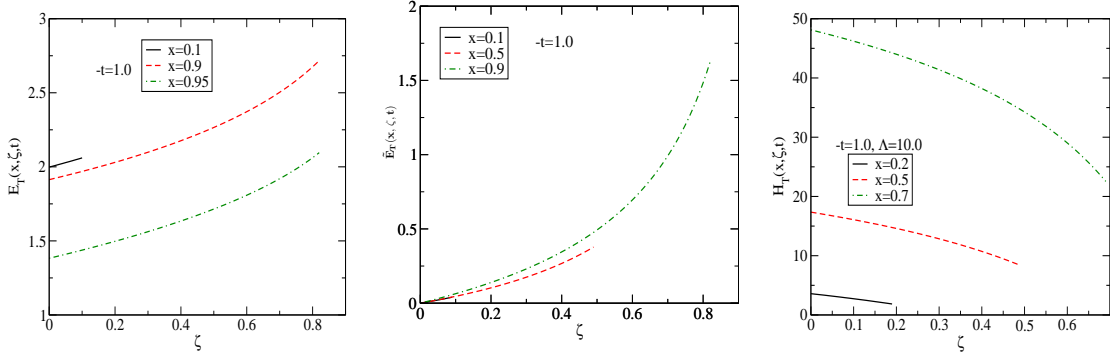
**Keywords:** DVCS, generalized parton distributions, chiral odd

**PACS:** 12.38.Bx, 12.38.Aw, 13.88.+e

## INTRODUCTION

At leading twist, there are three forward parton distributions (pdfs), namely, the unpolarized, helicity and transversity distributions. Similarly, three leading twist generalized quark distributions can be defined which in the forward limit, reduce to these three forward pdfs. The third one is chiral odd and is called the generalized transversity distribution  $F_T$ . This is defined as the off-forward matrix element of the bilocal tensor charge operator. It is parametrized in terms of four GPDs, namely  $H_T$ ,  $\tilde{H}_T$ ,  $E_T$  and  $\tilde{E}_T$  in the most general way [1, 2, 3]. The chiral-odd GPDs affect the transversely polarized quark distribution both in unpolarized and in transversely polarized nucleon in various ways. A relation for the transverse total angular momentum of the quarks has been proposed in [3], in analogy with Ji's relation, which involves a combination of second moments of  $H_T$ ,  $E_T$  and  $\tilde{H}_T$  in the forward limit.

In a previous work we have investigated the chiral odd GPDs for a simple spin-1/2 composite particle for  $\zeta = 0$  in impact parameter space [4]. In this work, using an overlap formula in the terms of the LFWFs both in the DGLAP ( $n \rightarrow n$ ) and ERBL ( $n+1 \rightarrow n-1$ ) regions [5], we investigate them in a simple model, namely for the quantum fluctuations of a lepton in QED at one-loop order [6]. We generalize this analysis by assigning a mass  $M$  to the external electrons and a different mass  $m$  to the internal electron lines and a mass  $\lambda$  to the internal photon lines with  $M < m + \lambda$  for stability. In effect, we shall represent a spin- $\frac{1}{2}$  system as a composite of a spin- $\frac{1}{2}$  fermion and a spin-1 vector boson [7, 8, 9, 10, 11]. This field theory inspired model has the correct correlation between the Fock components of the state as governed by the light-front eigenvalue equation, something that is extremely difficult to achieve in phenomenological models. GPDs in this model satisfy general properties like polynomiality and positivity. So it is interesting to investigate the general properties of GPDs in this model. By taking Fourier transform (FT) with respect to  $\Delta_\perp$ , we express the GPDs in transverse position space and by taking a FT with respect to  $\zeta$  we express them in longitudinal position space.



**FIGURE 1.** (Color online) Chiral-odd GPDs vs.  $\zeta$  for fixed  $t$  in  $\text{MeV}^2$  and different values of  $x$ .

### Chiral Odd GPDs using Overlap Formula

We use the parametrization of [3] for the chiral odd GPDs. Following [6, 7], we take a simple composite spin 1/2 state, namely an electron in QED at one loop to investigate the GPDs. The light-front Fock state wavefunctions corresponding to the quantum fluctuations of a physical electron can be systematically evaluated in QED perturbation theory. The state is expanded in Fock space and there are contributions from  $|e^- \gamma\rangle$  and  $|e^- e^- e^+\rangle$ , in addition to renormalizing the one-electron state. Both the two- and three-particle Fock state components are given in [12]. In the domain  $\zeta < x < 1$ , there are diagonal  $2 \rightarrow 2$  overlaps. Using the overlap formula in [5] we calculate the chiral odd GPDs in this kinematical region. They are given by;

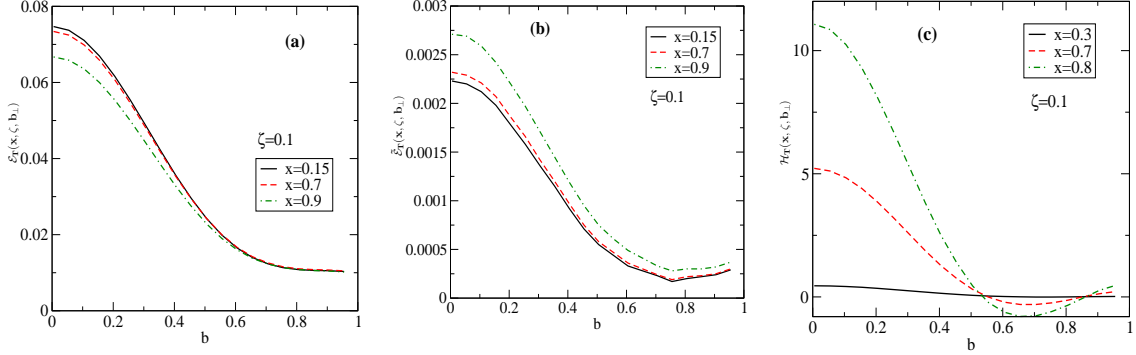
$$E_T(x, \zeta, t) = -\frac{e^2}{8\pi^3} \frac{2M\pi}{1-\zeta} \left(M - \frac{m}{x}\right) x(1-x) I_3, \quad (1)$$

$$\begin{aligned} \tilde{E}_T(x, \zeta, t) = & \frac{e^2}{8\pi^3} \frac{M\pi}{1-\zeta} \left[ -(1-x) \left\{ \left(M - \frac{m}{x}\right)x + \left(M - \frac{m}{x'}\right)x' \right\} I_1 \right. \\ & \left. + \left(M - \frac{m}{x}\right)x(1-x) I_2 \right], \end{aligned} \quad (2)$$

$$\begin{aligned} H_T(x, \zeta, t) = & \frac{e^2}{8\pi^3} \frac{\pi}{2} \left[ \frac{x+x'}{2(1-x)} \ln\left(\frac{\Lambda^4}{DD'}\right) + \left\{ \frac{x+x'}{2(1-x)} B(x, \zeta) + \frac{\zeta M}{1-\zeta} \left(M - \frac{m}{x}\right)x(1-x) \right\} I_2 \right. \\ & \left. - \frac{\zeta M}{1-\zeta} \left\{ \left(M - \frac{m}{x}\right)x(1-\zeta) + \left(M - \frac{m}{x'}\right)x' \right\} (1-x') I_1 \right], \end{aligned} \quad (3)$$

$$I_1 = \int_0^1 dy \frac{1-y}{Q(y)}, \quad (4)$$

$$I_2 = \int_0^1 dy \frac{1}{Q(y)}; \quad (5)$$



**FIGURE 2.** (Color online) Fourier spectrum of the chiral-odd GPDs vs.  $|b_\perp|$  for fixed  $\zeta$  and different values of  $x$

where  $Q(y) = y(1-y)(1-x')^2\Delta_\perp^2 - y(M^2x(1-x) - m^2(1-x) - \lambda^2x) - (1-y)(M^2x'(1-x') - m^2(1-x') - \lambda^2x')$  and

$$I_3 = \int_0^1 dy \frac{y}{Q(y)}. \quad (6)$$

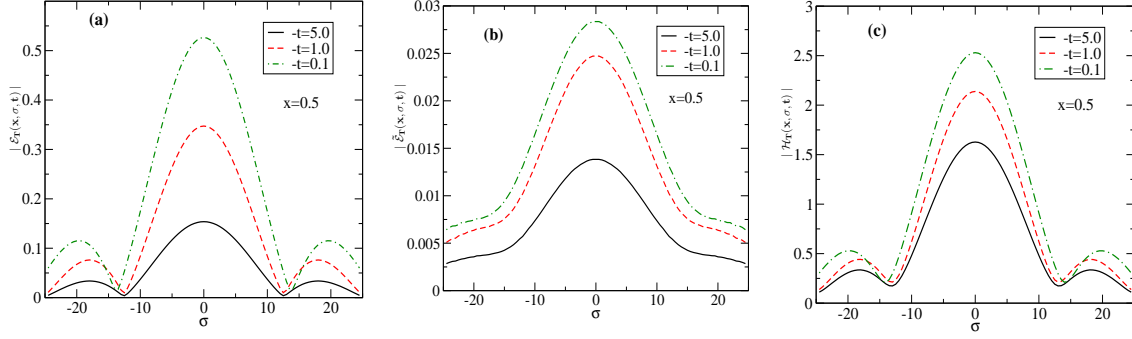
$D = M^2x(1-x) - m^2(1-x) - \lambda^2x$  and  $D' = M^2x'(1-x') - m^2(1-x') - \lambda^2x'$ .

In order to regulate the ultraviolet divergences we use a cutoff  $\Lambda$  on the transverse momentum  $k^\perp$ .  $\tilde{H}_T(x, \zeta, t)$  is zero in this model.

In Fig. 1 we have shown the chiral odd GPDs as functions of  $\zeta$  for fixed  $t$  and different values of  $x$ .

Introducing the Fourier conjugate  $b_\perp$  (impact parameter) of the transverse momentum transfer  $\Delta_\perp$ , the GPDs can be expressed in impact parameter space. Like the chiral even counterparts, chiral odd GPDs as well have interesting interpretation in impact parameter space [3]. In most experiments  $\zeta$  is nonzero, and it is of interest to investigate the chiral odd GPDs in  $b_\perp$  space with nonzero  $\zeta$ . The probability interpretation is no longer possible as now the transverse positions of the initial and final protons are different as there is a finite momentum transfer in the longitudinal direction. The GPDs in impact parameter space probe partons at transverse position  $|b_\perp|$  with the initial and final proton shifted by an amount of order  $\zeta |b_\perp|$ . Note that this is independent of  $x$  and even when GPDs are integrated over  $x$  in an amplitude, this information is still there [13]. Thus the chiral odd GPDs in impact parameter space gives the spin orbit correlations of partons in protons with their centers shifted with respect to each other. Taking the Fourier transform (FT) with respect to the transverse momentum transfer  $\Delta_\perp$  we get the GPDs in the transverse impact parameter space.

$$\begin{aligned} \mathcal{E}_T(x, \zeta, b_\perp) &= \frac{1}{(2\pi)^2} \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} E_T(x, \zeta, t) \\ &= \frac{1}{2\pi} \int \Delta d\Delta J_0(\Delta b) E_T(x, \zeta, t), \end{aligned} \quad (7)$$



**FIGURE 3.** (Color online) Fourier spectrum of the chiral-odd GPDs vs.  $\sigma$  for fixed  $x$  and different values of  $-t$  in  $\text{MeV}^2$ .

where  $\Delta = |\Delta_\perp|$  and  $b = |b_\perp|$ . The other impact parameter dependent GPDs  $\tilde{\mathcal{E}}_T(x, \zeta, b_\perp)$  and  $\mathcal{H}_T(x, \zeta, b_\perp)$  can also be defined in the same way. It is to be noted that  $\mathcal{H}_T(x, \zeta, b_\perp)$  for a free Dirac particle is expected to be a delta function; the smearing in  $|b_\perp|$  space is due to the spin correlation in the two-particle LFWFs. Fig. 2 shows the plots of the above three functions for fixed  $\zeta$  and different values of  $x$ . For given  $\zeta$ , the peak of  $\mathcal{H}_T(x, \zeta, b_\perp)$  as well as  $\tilde{\mathcal{E}}_T(x, \zeta, b_\perp)$  increases with increase of  $x$ , however for  $\mathcal{E}_T(x, \zeta, b_\perp)$  it decreases.

In [14], a phase space distribution of quarks and gluons in the proton is given in terms of the quantum mechanical Wigner distribution  $W(\vec{r}, \vec{p})$ , in the rest frame of the proton, which are functions of three position and three momentum coordinates. Wigner distributions are not accessible in experiment. However, if one integrates two momentum components one gets a reduced Wigner distribution  $W_\Gamma(\vec{r}, x)$  which is related to the GPDs by a Fourier transform. For given  $x$ , this gives a 3D position space picture of the partons inside the proton. If the probing wavelength is comparable to or smaller than the Compton wavelength  $\frac{1}{M}$ , where  $M$  is the mass of the proton, electron-positron pairs will be created, as a result, the static size of the system cannot be probed to a precision better than  $\frac{1}{M}$  in relativistic quantum theory. However, in light-front theory, transverse boosts are Galilean boosts which do not involve dynamics. So one can still express the GPDs in transverse position or impact parameter space and this picture is not spoiled by relativistic corrections. However, rotation involves dynamics here and rotational symmetry is lost. In [7], a longitudinal boost invariant impact parameter  $\sigma$  has been introduced which is conjugate to the longitudinal momentum transfer  $\zeta$ . It was shown that the DVCS amplitude expressed in terms of the variables  $\sigma, b_\perp$  show diffraction pattern analogous to diffractive scattering of a wave in optics where the distribution in  $\sigma$  measures the physical size of the scattering center in a 1-D system. In analogy with optics, it was concluded that the finite size of the  $\zeta$  integration of the FT acts as a slit of finite width and produces the diffraction pattern. We define a boost invariant impact parameter conjugate to the longitudinal momentum transfer as  $\sigma = \frac{1}{2}b^-P^+$  [7]. The chiral odd GPD  $E_T$  in

longitudinal position space is given by :

$$\begin{aligned}\mathcal{E}_T(x, \sigma, t) &= \frac{1}{2\pi} \int_0^{\zeta_f} d\zeta e^{i\frac{1}{2}P^+\zeta b^-} E_T(x, \zeta, t) \\ &= \frac{1}{2\pi} \int_0^{\zeta_f} d\zeta e^{i\sigma\zeta} E_T(x, \zeta, t).\end{aligned}\tag{8}$$

Similarly one can obtain  $\mathcal{H}_T(x, \sigma, t)$  and  $\tilde{\mathcal{E}}_T(x, \sigma, t)$  as well. Fig. 3 shows the plots of the Fourier spectrum chiral odd GPDs in longitudinal position space as a function of  $\sigma$  for fixed  $x = 0.5$  and different values of  $t$ . Both  $\mathcal{E}_T(x, \sigma, t)$  and  $\mathcal{H}_T(x, \sigma, t)$  show diffraction pattern as observed for the DVCS amplitude in [7]; the minima occur at the same values of  $\sigma$  in both cases. However  $\tilde{\mathcal{E}}_T(x, \sigma, t)$  does not show diffraction pattern. This is due to the distinctively different behaviour of  $\tilde{E}_T(x, \zeta, t)$  with  $\zeta$  compared to that of  $E_T(x, \zeta, t)$  and  $H_T(x, \zeta, t)$ .  $\tilde{E}_T(x, \zeta, t)$  rises smoothly from zero and has no flat plateau in  $\zeta$  and thus does not exhibit any diffraction pattern when Fourier transformed with respect to  $\zeta$ . The position of first minima in Fig.3 is determined by  $\zeta_f$ . For  $-t = 5.0$  and  $1.0$ ,  $\zeta_f \approx x = 0.5$  and thus the first minimum appears at the same position while for  $-t = 0.1$ ,  $\zeta_f = \zeta_{max} \approx 0.45$  and the minimum appears slightly shifted. This is analogous to the single slit optical diffraction pattern.  $\zeta_f$  here plays the role of the slit width. Since the positions of the minima(measured from the center of the diffraction pattern) are inversely proportional to the slit width, the minima move away from the center as the slit width (i.e.,  $\zeta_f$ ) decreases. The optical analogy of the diffraction pattern in  $\sigma$  space has been discussed in [7] in the context of DVCS amplitudes.

## CONCLUSION

In this work, we studied the chiral-odd GPDs in transverse and longitudinal position space. Working in light-front gauge, we used overlap formulas for the chiral odd GPDs in terms of proton light-front wave functions in the DGLAP region. We used a self consistent relativistic two-body model, namely the quantum fluctuation of an electron at one loop in QED. We used its most general form [6], where we have a different mass for the external electron and different masses for the internal electron and photon. The impact parameter space representations are obtained by taking Fourier transform of the GPDs with respect to the transverse momentum transfer. When  $\zeta$  is non-zero, the initial and final proton are displaced in the impact parameter space relative to each other by an amount proportional to  $\zeta$ . As this is the region probed by most experiments, it is of interest to investigate this. By taking a Fourier transform with respect to  $\zeta$  we presented the GPDs in the boost invariant longitudinal position space variable  $\sigma$ .  $H_T$  and  $E_T$  show diffraction pattern in  $\sigma$  space.

## ACKNOWLEDGMENTS

AM thanks the organizers of Spin 2008 for invitation and support and MHRD for support.

## REFERENCES

1. M. Diehl, Eur. Phys. J. C **19**, 485 (2001).
2. M. Diehl and P. Hagler, Eur.Phys.J.C **44**, 87 (2005).
3. M. Burkardt, Phys. Rev. **D 72**, 094020 (2005).
4. H. Dahiya, A. Mukherjee, Phys. Rev. **D 77**, 045032 (2008).
5. D. Chakrabarti, R. Manohar, A. Mukherjee; arXiv:0811.0521 [hep-ph].
6. S. J. Brodsky and S. D. Drell, Phys. Rev. **D 22**, 2236 (1980).
7. S. J. Brodsky, D. Chakrabarti, A. Harindranath, A. Mukherjee and J. P. Vary, Phys. Lett. B **641**, 440 (2006); Phys. Rev. **D 75**, 014003 (2007).
8. A. Harindranath, R. Kundu, W. M. Zhang, Phys. Rev. **D 59**, 094013 (1999); A. Harindranath, A. Mukherjee, R. Ratabole, Phys. Lett. **B 476**, 471 (2000); Phys. Rev. **D 63**, 045006 (2001).
9. D. Chakrabarti and A. Mukherjee, Phys. Rev. **D 71**, 014038 (2005).
10. D. Chakrabarti, A. Mukherjee, Phys. Rev. **D 72**, 034013 (2005).
11. A. Mukherjee and M. Vanderhaeghen, Phys. Lett. **B 542**, 245 Phys. Rev. **D 67**, 085020 (2003).
12. S. J. Brodsky, M. Diehl, D. S. Hwang, Nucl. Phys. **B 596**, 99 (2001); M. Diehl, T. Feldmann, R. Jacob, P. Kroll, Nucl. Phys. **B 596**, 33 (2001), Erratum-ibid **605**, 647 (2001).
13. M. Diehl, Eur.Phys.J.C**25**,223-232,2002, Erratum-ibid.C**31**, 277-278,2003.
14. X. Ji, Phy. Rev. Lett. **91**, 062001 (2003); A. Belitsky, X. Ji, F. Yuan, Phys. Rev. **D 69** 074014 (2006).